Multiplying factors correct power for ac waveforms

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The growing use of waveform generators, voltage-controlled oscillators, and multivibrators as signal sources means that engineers often have to measure currents and voltages in the form of rectangular, triangular, or sawtooth waves or pulse trains. (Conversion factors for voltmeter measurements on such waveforms were tabulated in *Electronics*, Aug. 30, 1973, p. 104.) The average power that one of these waveforms dissipates in a resistor (R) over an integral number of cycles is given by the root-mean-square voltage across the resistor (V_{rms}), the rms current through the resistor (I_{rms}), or both:

$$P = V_{rms}I_{rms}$$
$$= V_{rms}^2/R$$
$$= I_{rms}^2R$$

If measurements are made with meters that give true rms readings, the correct value for power can be calculated from the equations given above. But if the response of the ammeter or voltmeter is not truly rms, power values must be calculated from equations that contain a factor to correct for the meter response:

$$P = (V_mI_m) \times M$$
$$= (V_m^2/R) \times M$$
$$= (I_m^2R) \times M$$

In these equations, V_m and I_m are voltage and current values shown by the meters, and M is a multiplier that provides the correct value for power. Thus M is a combination of the conversion factor for meter response and the form factor for the waveform. Multiplier M is dimensionless.

The accompanying table shows values of M for various waveforms and various meters. For example, if a sawtooth voltage across a resistor is measured with a meter that responds to average voltage and is calibrated to rms for sine waves, then the power dissipated in the resistor is given by

$$P = (V_m^2/R) \times (32/3\pi^2)$$

For meters with a true rms response, M is always 1, so no column for true rms is included in the table.

If power is found from readings of both current and voltage meters, and the two meters have different responses, the power must be calculated from

$$P = V_mI_m(M_VM_I)^{1/2}$$

where M_V is the multiplier in the table that corresponds to the voltmeter response, and M_I is the multiplier that

MULTIPLIER (M) FOR POWER CALCULATION

Waveform	Ammeter or voltmeter response (see below)				
	I	II	III	IV	V
Sine:	1	1	$\pi^2/8$	1/2	1/8
Full-wave rectified sine:	1	1	$\pi^2/8$	1/2	1/2
Half-wave rectified sine:	2	1/2	$\pi^2/4$	1/4	1/4
Sine pulse:	T/t	t/T	$\pi^2T/8t$	t/2T	t/2T
Segmental sine:	4E/C	E	$\pi^2E/2C$	E/2	E/8
Full-wave rectified segmental sine:	4E/C	E	$\pi^2E/2C$	E/2	E/2
Half-wave rectified segmental sine:	8E/C	E/2	π^2E/C	E/4	E/4
Sine squared:	$12T/\pi^2t$	3t/4T	3T/2t	3t/8T	3t/8T
Fractional sine pulse:	$4B/\pi A$	B/π	$\pi B/2A$	$B/2\pi$	$B/2\pi$
Triangle or sawtooth:	$32/3\pi^2$	2/3	4/3	1/3	1/12
Full-wave rectified triangle or sawtooth:	$32/3\pi^2$	2/3	4/3	1/3	1/3
Half-wave rectified triangle or sawtooth:	$64/3\pi^2$	1/3	8/3	1/6	1/6
Triangle or sawtooth pulse:	$32T/3\pi^2t$	2t/3T	4T/3t	t/3T	t/3T
Square:	$8/\pi^2$	2	1	1	1/4
Dc and full-wave rectified square:	$8/\pi^2$	2	1	1	1
Half-wave rectified square:	$16/\pi^2$	1	2	1/2	1/2
Rectangular pulse:	$8T/\pi^2t$	2t/T	T/t	t/T	t/T
Exponential pulse (critically damped)	$2T/\pi^2e^2t$	$e^2t/2T$	$T/4e^2t$	$e^2t/4T$	$e^2t/4T$

$A = [(\sin\alpha - \alpha\cos\alpha)/(1 - \cos\alpha)]^2$
$B = [2\alpha + \alpha\cos 2\alpha - (3/2)\sin\alpha]/(1 - \cos\alpha)^2$
$\alpha \equiv \pi t/T$ radians

$C = (1 - \cos\theta)^2$
$E = (\pi\theta/180) - (\sin 2\theta/2)$
θ = Conduction angle (degrees)

$e = 2.71828\ldots$ $\pi = 3.14159\ldots$

Ammeter or voltmeter response

I = Average-responding, calibrated rms for sines
II = Peak-responding, calibrated rms for sines
III = True average
IV = True peak
V = Peak-to-peak

corresponds to the ammeter used in the measurement.

The accuracy of some of these correction factors depends on how nearly the actual waveform approaches the ideal. Also, most ac meters do not give accurate readings for frequencies below 10 or 20 hertz, and they do not give any indication for dc. Thus the full-wave-rectified square wave may produce zero readings, depending upon the meter used. □